Received: 15 March 2019 | Revised: 1 January 2020 | Accepted: 17 January 2020

DOI: 10.1002/zamm.201900074

ORIGINAL PAPER

Waves in nonlocal thermoelastic solids of type III

Nihar Sarkar[1] | Mitali Bachher[2] | Narayan Das[3] | Soumen De[4] | Nantu Sarkar[4]

[1]Department of Mathematics, City College, Kolkata 700009, India

[2]Department of Mathematics, Dinabandhu Mahavidyalaya, Bongaon, West Bengal 743235, India

[3]Government General Degree College, Dantan-II, Paschim Medinipur, Medinipur, West Bengal 721445, India

[4]Department of Applied Mathematics, University of Calcutta, Kolkata 700009, India

Correspondence
Nantu Sarkar, Department of Applied Mathematics, University of Calcutta, Kolkata 700009, India.
Email: nsarkarindian@gmail.com

In this paper, the propagation of time-harmonic thermoelastic plane waves is studied in an infinite nonlocal elastic continuum. The type III Green-Naghdi model (with energy dissipation) of generalized thermoelasticity and the Eringen's nonlocal elasticity model are adopted to address this problem. We found two sets of the coupled longitudinal waves which are dispersive in nature and experience attenuation. In addition to the coupled waves, there also exists one independent vertically shear-type wave which is dispersive but experiences no attenuation. All these waves are found to be influenced by the elastic nonlocality parameter. Furthermore, the shear-type wave is found to face a critical frequency, while the coupled longitudinal waves may face critical frequencies conditionally. Reflection phenomenon of an incident coupled longitudinal waves from a rigid and thermally insulated boundary surface of a homogeneous and isotropic nonlocal thermoelastic half-space is investigated. Using these boundary conditions, the formulae for various reflection coefficients and their respective energy ratios are presented. For a particular model, various graphs are plotted to analyze the behavior of the phase speeds, reflection coefficients and their respective energy ratios. The amplitude ratios of the reflected waves and their respective energy ratios are determined analytically. For a particular model, the effect of elastic nonlocality parameter on the variations of phase speeds, attenuation coefficients, amplitude ratios and corresponding energy ratios of the reflected waves are presented graphically. Finally, analysis of the various results have been interpreted.

KEYWORDS
dispersion, energy partition, green-nagdhi model, nonlocal, reflection

1 | INTRODUCTION

Stress tensor at any reference point in a nonlocal continuum depends not only on the strain at that point but also on the strain at all other points of the continuum [1]. This observation is in accordance with the atomic theory of lattice dynamics and experimental observations on phonon dispersion. In the limiting case, when the effect of strain at points other than x are ignored, one can recover the classical (local) elasticity. In the year 1971, Edelen and Laws [2] developed the thermodynamics of systems with nonlocality. In the same year Edelen et al. [3] introduced nonlocal continuum mechanics. Following these pioneer works, Eringen and Edelen [4] formulated nonlocal elasticity in 1972. Uniqueness in the linear theory of nonlocal elasticity was proved by Altan [5]. Chirita [6] investigated some boundary value problems in the context of the theory of nonlocal elasticity. Narendra and Gopalakrishnan [7] studied ultrasonic wave characteristics of nanorods via nonlocal strain gradient models. They also studied the nonlocal scale effect of ultrasonic wave characteristics of nanorods [8]. Narendar et al. [9] investigated the prediction of nonlocal scaling parameter for armchair and zigzag single-walled carbon nanotubes based on molecular structural mechanics, nonlocal elasticity and wave propagation. Narendra [10] formulated spectral finite element and nonlocal continuum mechanics based formulation to study torsional wave propagation in nanorods. Malagu et al. [11] studied one-dimensional nonlocal elasticity for tensile single-walled carbon nanotubes through a molecular structural mechanics characterization. Khurana and Tomar [12] extended the theory of nonlocal elasticity to nonlocal theory of microstretch elasticity and then they investigated wave propagation in nonlocal microstretch solid. Sing [13] studied wave propagation in nonlocal elastic solid with voids.

The theory of nonlocal elasticity has been extended to nonlocal thermoelasticity by many researchers during the last five decades. Eringen [14] first proposed the theory of nonlocal thermoelasticity in the year 1974. He summarized the balance laws and entropy inequality obtained by himself in the theory of nonlocal elasticity. He derived the constitutive relations and some nonlocal moduli of the nonlocal thermoelasticity based on two basic constitutive axioms proposed by himself. After three years of Eringen's nonlocal theory of thermoelasticity, Balta and Suhubi [15] extended this theory to the theory of nonlocal generalized thermoelasticity within the framework of nonlocal continuum mechanics. They derived the constitutive relations through the systematic use of the nonlocal version of the generalized thermodynamics. The constitutive relations are linearized and field equations are provided for homogeneous isotropic solids. They also studied thermal waves in rigid conductors using the theory of nonlocal generalized thermoelasticity. Yu et al. [16] proposed size-dependent generalized thermoelasticity using Eringen's nonlocal model. Yu et al. [17] also introduced nonlocal thermoelasticity based on nonlocal heat conduction and nonlocal elasticity. They also applied these theories [16, 17] to study some one-dimensional problems. Bachher and Sarkar [18] extended the nonlocal theory of elasticity with voids [13] to the nonlocal theory of thermoelastic materials with voids and fractional derivative heat transfer. They also applied this new theory to study the one-dimensional transient response of a thermoelastic nonlocal infinite medium with voids. Li et al. [19] investigated the reflection and transmission of elastic waves at an interface with consideration of couple stress and thermal wave effects. Recently, Sarkar and Tomar [20] studied plane waves in nonlocal thermoelastic solid with voids and thermal relaxation time and Mondal and Sarkar [21] reported waves in dual-phase-lag thermoelastic materials with voids based on Eringen's nonlocal elasticity.

Jeffreys [22] considered the problems of reflection of plane harmonic waves at a solid half-space. Gutenberg [23] obtained energy relation of reflected and refracted seismic waves. Reflection and refraction of elastic waves with seismological applications has been done by Knott [24]. Beevers and Bree [25] discussed wave reflection problems in linear thermoelasticity. Sinha and Sinha [26] and Sinha and Elsibai [27] discussed the reflection of the thermoelastic waves from the free surface of a solid half-space and at the interface of two semi infinite media in welded contact in the context of generalized thermoelasticity. Sharma et al. [28] investigated the problem of thermoelastic wave reflection from the thermally insulated and isothermal stress-free as well as rigidly fixed boundaries of a solid half-space in the context of different linear theories of generalized thermoelasticity vz. Green-Nagdhi (GN-II) [29], Lord-Shulman (LS) [30] and Green-Lindsay (GL) [31] theories. The angles of incidence and reflection of P- and SV- waves with normal to the half-space have not been considered in their works. Das et al. [32] studied the problem of reflection of thermoelastic wave from a thermally insulated and isothermal stress-free boundary of a solid half-space in the context of Green-Naghdi model II [29]. They preferred to investigate the problem by specifying the angle of incidence and reflection with the normal to the half-space. Kumar et al. [33] studied thermomechanical interactions in a transversely isotropic magneto-thermoelastic with and without energy dissipation with combined effects of rotation, vacuum and two temperatures. Lata [34, 35] discussed the reflection and refraction of plane waves in layered nonlocal elastic and anisotropic thermoelastic medium and effect of energy dissipation on plane waves in sandwiched layered thermoelastic medium. Othman and Song [36–39] studied several problems on reflection of thermoelastic waves under different conditions.

The present contribution is concerned with the propagation of time-harmonic thermoelastic plane waves is studied in an infinite nonlocal elastic continuum. The type III Green-Naghdi model [40] (with energy dissipation) of generalized thermoelasticity and the Eringen's nonlocal elasticity [1, 4] model are adopted to address this problem. We found two sets of the coupled longitudinal waves which are dispersive in nature and experience attenuation. In addition to the coupled waves, there also exists one independent vertically shear-type wave which is dispersive but experiences no attenuation. All these waves are found to be influenced by the elastic nonlocality parameter. Furthermore, the shear-type wave is found to face a critical frequency, while the coupled longitudinal waves may face critical frequencies conditionally. Reflection phenomenon of an incident coupled longitudinal waves from a rigid and thermally insulated boundary surface of a homogeneous and isotropic nonlocal thermoelastic half-space is investigated. Using these boundary conditions, the formulae for various reflection coefficients and their respective energy ratios are presented. For a particular model, various graphs are plotted to analyze the behavior of the phase speeds, reflection coefficients and their respective energy ratios. The amplitude ratios of the reflected waves and their respective energy ratios are determined analytically. For a particular model, the effect of elastic nonlocality parameter on the variations of phase speeds, attenuation coefficients, amplitude ratios and corresponding energy ratios of the reflected waves are presented graphically. Finally, analysis of the various results have been interpreted.

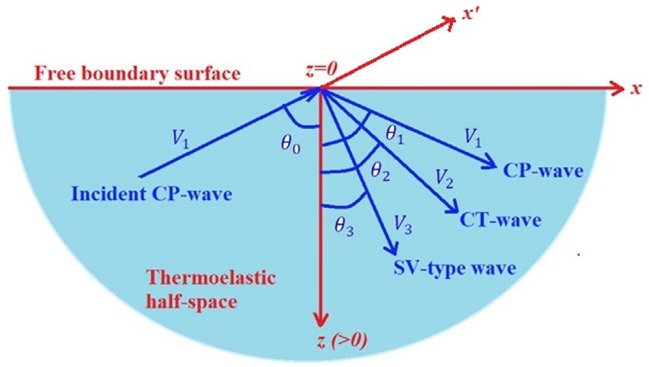

FIGURE 1 Incident and reflected thermoelastic waves at the surface $z = 0$

2 | GOVERNING EQUATIONS AND FORMULATION OF THE PROBLEM

We consider a linear, homogenous and isotropic nonlocal thermoelastic solid in a rectangular Cartesian coordinate system $Oxyz$, initially un-deformed and at uniform temperature T_0. Let the origin of the coordinate system $Oxyz$ be fixed at any point on the boundary surface $z = 0$ of the half-space $z \geq 0$ with $z-$ axis directed normally inside the medium and $x-$axis is along the horizontal direction. The $y-$axis is taken in the direction of the line of intersection of the plane wavefront with the plane surface. A schematic diagram of the present problem is illustrated in Figure 1.

For a two-dimensional plain strain state parallel to the $x - z$ plane, all the field variables may be dependent on x, z and t only. So, the displacement vector $\vec{u}$ and the temperature field Θ may take the forms

$$\vec{u}(x, z, t) = (u,\ 0,\ w), \quad \Theta = \Theta(x, z, t).$$

The basic governing equations for a linear, homogeneous, isotropic, nonlocal thermoelastic solid of type III with zero body force and without considering heat source are read from Das et al. [42] and Sarkar et al. [43] as follows:

$$\mu\nabla^2\vec{u} + (\lambda + \mu)\vec{\nabla}(\vec{\nabla}\cdot\vec{u}) - \gamma\vec{\nabla}\Theta = \rho(1 - \epsilon^2\nabla^2)\ddot{\vec{u}}, \tag{1}$$

$$\left(K^* + K_\Theta\frac{\partial}{\partial t}\right)\nabla^2\Theta = \rho C_E\ddot{\Theta} + \gamma T_0\vec{\nabla}\cdot\ddot{\vec{u}}, \tag{2}$$

$$(1 - \epsilon^2\nabla^2)\vec{\tau} = \vec{\tau}^L = \mu\left(\vec{\nabla}\vec{u} + \vec{\nabla}\vec{u}^T\right) + \left(\lambda\vec{\nabla}\cdot\vec{u} - \gamma\Theta\right)\delta_{ij}\vec{I}, \tag{3}$$

where $\nabla^2 \equiv \partial^2/\partial x^2 + \partial^2/\partial z^2$, ϵ $(= e_0 a)$ is the elastic nonlocal parameter [4, 13] having the dimension of length, a and e_0, respectively are an internal characteristic length and a constant, $\vec{\tau}$ is the stress tensor, $\vec{\tau}^L$ stands for stress tensor in local thermoelastic medium, $\lambda,\ \mu$ are Lame's constants, $\gamma = (3\lambda + 2\mu)\alpha_T$ is the thermoelastic coupling constant, α_T is the coefficient of volume expansion, Θ is the temperature change over the initial reference temperature T_0 of the medium, δ_{ij} is the Kronecker delta, ρ is the mass density, $K^*(> 0)$ is a material constant characteristic of the medium, K_Θ is the thermal conductivity, C_E is the specific heat at constant strain, $\vec{I}$ is the identity tensor and $i, j = x, z$. Note that in the above equations, a comma followed by a suffix denotes a spatial derivative, a superposed dot stands for a time-differentiation and an upper arrow represents a vector quantity.

The system of Equations (1) and (2) is fully hyperbolic in nature and as such both the elastic and thermal waves propagate with finite speed. Equations (1)–(3) describe the nonlocal generalized thermoelasticity theory based on the Green-Naghdi theory with energy dissipation [40] that will be used in the sequel.

In order to make Equations (1)–(3) dimensionless, the following quantities are introduced

$$\left(x', z', \epsilon'\right) = \frac{1}{l}(x, z, \epsilon),\ \left(u', w'\right) = \frac{(\lambda + 2\mu)}{\gamma l T_0}(u, w),\ t' = \frac{v}{l}t,\ \Theta' = \frac{\Theta}{T_0},\ \tau'_{ij} = \frac{\tau_{ij}}{\gamma T_0}, \tag{4}$$

where l is a standard length and v is a standard speed. Inserting (4) into the Equations (1)–(3) and suppressing the primes, we can write

$$c_s^2\nabla^2\vec{u} + (c_p^2 - c_s^2)\vec{\nabla}(\vec{\nabla}\cdot\vec{u}) - c_p^2\vec{\nabla}\Theta = (1 - \epsilon^2\nabla^2)\ddot{\vec{u}}, \tag{5}$$

$$\left(c_\Theta^2 + \bar{K}\frac{\partial}{\partial t}\right)\nabla^2\Theta - \ddot{\Theta} - \epsilon_\Theta \vec{\nabla}\cdot\ddot{\vec{u}} = 0, \tag{6}$$

$$(1-\varepsilon^2\nabla^2)\vec{\tau} = \vec{\tau}^L = \left(1-2\delta^2\right)(\vec{\nabla}\cdot\vec{u})\delta_{ij}\vec{I} + \delta^2\left(\vec{\nabla}\vec{u} + \vec{\nabla}\vec{u}^T\right) - \Theta\delta_{ij}\vec{I}, \tag{7}$$

where

$$\delta = \frac{c_s}{c_p}, c_p^2 = \frac{\lambda+2\mu}{\rho\upsilon^2},\quad c_s^2 = \frac{\mu}{\rho\upsilon^2},\quad c_\Theta^2 = \frac{K^*}{\rho C_E\upsilon^2},\quad \bar{K} = \frac{K_\Theta}{\rho C_E\upsilon l},\quad \epsilon_\Theta = \frac{\gamma^2 T_0}{\rho C_E(\lambda+2\mu)}.$$

Here, ϵ_Θ is defined as the dimensionless thermoelastic coupling constant. We now introduce the scalar and vector potentials, namely ϕ and $\vec{\psi}$ respectively through the Helmholtz vector decomposition technique as

$$\vec{u} = \vec{\nabla}\phi + \vec{\nabla}\times\vec{\psi},\qquad \vec{\nabla}\cdot\vec{\psi} = \vec{0}, \tag{8}$$

Plugging Equation (8) into Equations (5)–(7), we obtain

$$c_p^2\nabla^2\phi - (1-\varepsilon^2\nabla^2)\ddot{\phi} = c_p^2\Theta, \tag{9}$$

$$c_s^2\nabla^2\vec{\psi} - (1-\varepsilon^2\nabla^2)\ddot{\vec{\psi}} = \vec{0}, \tag{10}$$

$$\left(c_\Theta^2 + \bar{K}\frac{\partial}{\partial t}\right)\nabla^2\Theta - \ddot{\Theta} - \epsilon_\Theta\nabla^2\ddot{\phi} = 0. \tag{11}$$

We note that the temperature field Θ is coupled with the potential ϕ. Thus, Equations (9) and (11) together create two sets of coupled longitudinal thermal-elastic wave. We also observed from Equation (10) that the potential $\vec{\psi}$ is uncoupled with Θ and ϕ, where

$$\vec{\psi} = (0, \psi, 0). \tag{12}$$

Choosing (12), the potential $\vec{\psi}$ corresponds to the displacement motion in the $x-z$ plane due to a vertically shear type (SV-type) wave, governed by the equation

$$c_s^2\nabla^2\psi = (1-\varepsilon^2\nabla^2)\ddot{\psi}. \tag{13}$$

Hence, we conclude that the thermal wave effect has no influence on the SV-type wave propagation in the case of nonlocal thermoelasticity of type III.

3 | DISPERSION RELATION AND ITS SOLUTIONS

For a plane harmonic wave propagating in the $x'-$direction with wave speed c (see Figure 1), we can take

$$(\phi, \Theta) = (A_\phi, A_\Theta)\exp\{\imath(kx' - \omega t)\}, \tag{14}$$

where A_ϕ, A_Θ are constants representing the wave amplitudes, $\imath = \sqrt{-1}$, k is the dimensionless complex wavenumber and ω $(= kc)$ is the dimensionless assigned real angular frequency.

From Figure 1, x' is obtained as $x' = x\sin\theta_0 - z\cos\theta_0$, where θ_0 is the angle of incidence of the incident wave with the normal to the surface $z = 0$. Hence, Equation (14) becomes

$$(\phi, \Theta) = (A_\phi, A_\Theta)\exp\{\imath k(x\sin\theta_0 - z\cos\theta_0) - \imath\omega t\}. \tag{15}$$

Inserting Equation (15) into Equations (9) and (11), we get

$$\left[k^2(c_p^2 - \varepsilon^2\omega^2) - \omega^2\right]A_\phi + c_p^2 A_\Theta = 0, \tag{16}$$

$$-\epsilon_\Theta \omega^2 k^2 A_\phi + \left[\omega^2 - k^2\left(c_\Theta^2 - \imath\omega\bar{K}\right)\right]A_\Theta = 0. \tag{17}$$

For non-trivial solutions of the system of Equations (16) and (17) for the unknowns A_ϕ and A_Θ, the coefficient matrix must be singular, which leads to

$$c^4 - L_1 c^2 + L_2 = 0, \tag{18}$$

where

$$L_1 = (1+\epsilon_\Theta)c_p^2 + \left(c_\Theta^2 - \imath\omega\bar{K}\right) - \varepsilon^2\omega^2, \;\; L_2 = \left(c_\Theta^2 - \imath\omega\bar{K}\right)\left(c_p^2 - \varepsilon^2\omega^2\right). \tag{19}$$

The quadratic Equation (18) in c^2 is the general dispersion relation for the wave propagation in nonlocal thermoelastic solids of type III. Note that the coefficient L_1 is complex while L_2 is real for $\omega > 0$. The roots of the dispersion relation (18) are

$$c_{1,2}^2 = \frac{1}{2}\left[L_1 \pm \sqrt{L_1^2 - 4L_2}\right]. \tag{20}$$

Here c_1^2 corresponds to '+' sign and c_2^2 corresponds to '-' sign. Corresponding to these roots, there exist two sets of coupled longitudinal waves, namely, a coupled elastic wave ($CP-$wave) and a coupled thermal wave ($CT-$wave) whose phase speeds V_j and attenuation coefficients Q_j are given by (cf. Sarkar et al. [43], Achenbach [44])

$$V_j = \frac{\left(\Re(c_j)\right)^2 + \left(\Im(c_j)\right)^2}{\Re(c_j)}, \quad Q_j = \frac{-\omega\Im(c_j)}{\left(\Re(c_j)\right)^2 + \left(\Im(c_j)\right)^2}, \tag{21}$$

where $\Re(\cdot)$ and $\Im(\cdot)$ denote the real and imaginary parts. Since these roots are complex, the coupled longitudinal waves are attenuating. Also, the CP- and the CT-waves are dispersive in nature.

3.1 | Identification of the phase speeds of CP- and CT-waves

For uncoupled local thermoelasticity (UCT) ($\epsilon_\Theta = 0,\ \varepsilon = 0$), the roots c_1 and c_2 become

$$c_1 = c_p, \quad c_2 = \sqrt{c_\Theta^2 - \imath\omega\bar{K}}.$$

Thus, we conclude that for the present problem ($\epsilon_\Theta \neq 0$ and $\varepsilon \neq 0$), while V_1 represents the speed of a $CP-$wave, V_2 represents the speed of a $CT-$wave (according to our consideration of the sign of c_1^2 and c_2^2).

Proceeding in the same way as above, we may seek the solution for the $SV-$type wave governed by the Equation (13) as

$$\psi = A_\psi \exp\{\imath k_3(x\sin\theta_0 - z\cos\theta_0) - \imath\omega t\}, \tag{22}$$

where A_ψ is the amplitude of the wave, k_3 and c_3 are the wavenumber and the phase speed, respectively of the SV-type wave in the nonlocal medium. Using (22) in (13), we get

$$c_3 = \sqrt{c_s^2 - \epsilon^2\omega^2}. \tag{23}$$

The above result shows that the classical transverse wave speed, c_s is reduced in the nonlocal thermoelastic medium of type III due to the presence of the elastic nonlocality. The expression (23) also shows that the SV-type wave is weakly dispersive in case of nonlocal thermoelasticity of type III in contrast to the local thermoelasticity of type III. The phase speed, V_3 and the attenuation coefficient, Q_3 of the SV-type wave can be obtained by using the same formulae as noted in (21).

3.2 | Further discussions

Here we shall investigate the frequency range of CP-, CT- and SV-type waves in the nonlocal thermoelastic solid of type III as follows:

(i) To find out the frequency range for the SV-type wave traveling with speed c_3, we see from (23) that $c_3 = 0$ at $\omega = \omega_{c_3} = c_s/\varepsilon$ provided $\varepsilon \neq 0$. For $\omega > \omega_{c_3}$, we observe that the speed c_3 will be purely imaginary and that generates a standing damped wave in time whose amplitude decays exponentially with time t. Thus we can declare that this wave is a propagating wave in the frequency range $0 < \omega < \omega_{c_3}$. Beyond $\omega = \omega_{c_3}$, the wave is no more a propagating wave. Here $\omega = \omega_{c_3}$ act as the *critical frequency* for the SV-type wave.

(ii) Now, we determine the frequency range for the CP- and CT-waves. The coefficient L_2 in (19) vanishes at $\omega = \omega_{c_1} = c_p/\varepsilon$ provided $\varepsilon \neq 0$. It can also be verified that $\Re(L_1) > 0$ and $\Im(L_1) < 0$ at the frequency $\omega = \omega_{c_2}$. At $\omega = \omega_{c_2}$,

$$c_1^2 = L_1(\omega = \omega_{c_2}) = \epsilon_\Theta c_p^2 + c_\Theta^2 - \imath\omega\bar{K} \quad \text{and} \quad c_2^2 = 0,$$

which in turn means that the CT-wave disappears while the CP-wave travels whose amplitude decays exponentially with time.

(iii) In the entire range of frequency ($0 < \omega < \infty$), the values of V_j and Q_j for the CP- and CT-waves are given by (21). In the entire range of frequency, both of the CP- and CT-waves propagate except at the frequency $\omega = \omega_{c_2}$, where only one of the waves can travel and the other one disappears. Similar to the SV-type wave, we can say that the CT-wave is a propagating wave for $0 < \omega < \omega_{c_2}$ and is no more a propagating wave for $\omega \geq \omega_{c_2}$. For $\omega \geq \omega_{c_2}$, the CT-wave represents merely a distance decaying vibrations. Thus $\omega = \omega_{c_2}$ act as a *critical frequency* for the CT-wave.

3.3 | Nature of the wave speeds of the propagating waves at high and low frequencies

In this subsection, we wish to look at the behavior of the phase speeds of the different waves at high and low frequencies.

3.3.1 | Low frequency ($\omega \to 0$)

From Equation (20), we obtain the limiting values of c_1 and c_2 at low frequency as,

$$c_{1,2}^2 = \frac{(1+\epsilon_\Theta)c_p^2 + c_\Theta^2 \pm \sqrt{\left((1+\epsilon_\Theta)c_p^2 - c_\Theta^2\right)^2 + 4\epsilon_\Theta c_p^2 c_s^2}}{2}. \tag{24}$$

For UCT ($\epsilon_\Theta = 0$), we have $c_1 = c_p$ and $c_2 = c_\Theta$. So, it is interesting to note that the wave speeds are independent of the elastic nonlocality present in the medium at low frequency. Equation (23) shows that at low frequency, $c_3 = c_s$, i.e., the wave speed c_3 of the $SV-$type wave remains the same as to the speed c_s of the classical shear wave.

3.3.2 | High frequency ($\omega \to \infty$)

We can notice from Equation (23) that the speed of the SV-type wave, c_3 decreases first with an increase in ω and goes to zero at $\omega = \omega_{c_3}$. Beyond $\omega = \omega_{c_3}$, the speed c_3 becomes purely imaginary and the absolute value of c_3 goes on increasing with a further increase of ω. Equation (20) exhibits that beyond $\omega = \omega_{c_1}$, the absolute values of c_j $(j = 1, 2)$ increase with further increase of the frequency ω.

3.4 | Special cases

3.4.1 | Nonlocal generalized thermoelasticity of type II

For $\bar{K} \ll c_\Theta^2$ (i.e. at a very low thermal conductivity), we may neglect $\bar{K}$ and obtain the following expressions for the roots $c_{1,2}$:

$$c_{1,2}^2 = \frac{1}{2}\left[(1+\epsilon_\Theta)c_l^2 + c_\Theta^2 - \varepsilon^2\omega^2 \pm \sqrt{\Delta}\right], \tag{25}$$

where

$$\Delta = \left[(1+\epsilon_\Theta)c_p^2 - c_t^2 - \varepsilon^2\omega^2\right]^2 + 4\epsilon_\Theta c_p^2(c_\Theta^2 + \varepsilon^2\omega^2) + 4\varepsilon^2\omega^2 c_\Theta^2.$$

The expression in (25) tells us that the CP- and CT-waves are dispersive in nature and experience attenuation in the case of nonlocal medium. If we neglect the elastic nonlocality from the medium ($\varepsilon = 0$), then we can recover all the results of Chandrasekharaiah [41] (in absence of rotational effect).

3.4.2 | Thermoelastic undamped waves without energy dissipation (Green-Naghdi theory of type II) [29]

If the elastic nonlocality is absent from the medium, then at a very low thermal conductivity ($\bar{K} \ll c_\Theta^2$), the dispersion relation (18) is simplified to

$$c_p^2 c_\Theta^2 k^4 - k^2\omega^2\left[(1+\epsilon_\Theta)c_l^2 + c_\Theta^2\right] + \omega^4 = 0, \tag{26}$$

which was earlier obtained by Das et al. [32]. The roots of (26) are

$$k_{1,2}^2 = \frac{\omega^2}{2c_p^2 c_\Theta^2}\left[(1+\epsilon_\Theta)c_l^2 + c_\Theta^2 \mp \sqrt{\left\{(1+\epsilon_\Theta)c_l^2 - c_\Theta^2\right\}^2 + 4\epsilon_\Theta c_l^2 c_\Theta^2}\right]. \tag{27}$$

The expressions in (27) are exactly same as in Das et al. [32]. In this case, k_j $(j = 1, 2)$ are purely real, and therefore these roots provide the wavenumber of two non-dispersive and non-attenuating coupled longitudinal waves (CP- and CT-waves) which are in complete agreement with the results reported by Das et al. [32]. Similarly, putting $\varepsilon = 0$ into (23), we see that the $SV-$type wave speed c_3 in the local thermoelastic medium of type II reduces to the classical SV-wave speed, c_s [32].

3.4.3 | Nonlocal elastic solid

If we neglect the thermal wave effect, then we shall be left with a nonlocal elastic medium only. In this case, the speeds of the coupled longitudinal waves reduce to $c_1 = \sqrt{c_p^2 - \varepsilon^2\omega^2}$ and $c_2 = 0$. The speed of the SV-type wave remains the same as $c_3 = \sqrt{c_\Theta^2 - \varepsilon^2\omega^2}$, since this wave is independent of the thermal field. It is quite interesting to note that in a nonlocal elastic medium, the square of the speeds of the classical longitudinal wave (P-wave) as well as the classical shear wave (SV-wave) are frequency dependent (dispersive) and both are reduced by an amount equal to $\varepsilon^2\omega^2$, a result earlier obtained by [13] in the relevant medium. The frequencies $\omega = \omega_{c_j}$ $(j = 1, 3)$ act as the critical frequencies for the respective waves, beyond which the waves are no more propagating waves.

3.4.4 | Uncoupled nonlocal thermoelasticity

In this theory, we obtain the roots c_1 and c_2 as

$$c_1 = \sqrt{c_p^2 - \varepsilon^2\omega^2},\ c_2 = \sqrt{c_\Theta^2 - \iota\omega\bar{K}}. \tag{28}$$

From the above expressions, one can easily observe that the elastic wave and the thermal wave are dispersive and experience attenuation in contrast to the uncoupled local thermoelasticity. We also note that, in case of uncoupled local thermoelasticity, the elastic wave is non-attenuating as well as non-dispersive while the thermal wave is dispersive and experiences attenuation.

4 | REFLECTION OF THERMOELASTIC WAVES FROM A THERMALLY INSULATED RIGID SURFACE

Let a train of CP-wave having amplitude A_0 and phase speed V_1 is made incident on the surface $z = 0$ making an angle θ_0 with the normal to the free surface $z = 0$ as shown in Figure 1. Assuming that the radiation in a vacuum is neglected, when it impinges the boundary $z = 0$, three reflection waves in the medium are created. Suppose the reflected CP-, CT- and SV-type waves make angles θ_1, θ_2 and θ_3, respectively with the positive $z-$axis. Then, the complete structures of the wave fields consisting of the incident and reflected waves may be expressed as

$$\phi = A_0 \exp\left\{\iota k_1(x\sin\theta_0 - z\cos\theta_0) - \iota\omega t\right\} + \sum_{j=1}^{2} A_j \exp\left\{\iota k_j(x\sin\theta_j + z\cos\theta_j) - \iota\omega t\right\}, \tag{29}$$

$$\Theta = \zeta_1 A_0 \exp\left\{\imath k_1 (x \sin\theta_0 - z\cos\theta_0) - \imath\omega t\right\} + \sum_{j=1}^{2} \zeta_j A_j \exp\left\{\imath k_j (x\sin\theta_j + z\cos\theta_j) - \imath\omega t\right\}, \tag{30}$$

$$\psi = B_1 \exp\left\{\imath k_3 (x\sin\theta_3 + z\cos\theta_3) - \imath\omega t\right\}, \tag{31}$$

where A_1, A_2 and B_1 represent the coefficients of amplitudes of the reflected CP-, CT- and SV-type waves, respectively, and

$$\zeta_j = \frac{\omega^2}{c_p^2 c_j^2}\left(c_j^2 - c_p^2 + \varepsilon^2\omega^2\right), \quad j = 1, 2. \tag{32}$$

The amplitude ratios of the reflected waves are defined as the ratios of the amplitudes of the reflected waves to the amplitude of the incident wave and are determined by the well-defined boundary conditions on the surface $z = 0$.

4.1 | Boundary conditions

We assume the surface $z = 0$ to be rigidly fixed and thermally insulated. These can be expressed mathematically as

$$u = w = \frac{\partial\Theta}{\partial z} = 0, \quad \text{at} \quad z = 0. \tag{33}$$

In terms of the potential functions ϕ and ψ, the rigid boundary conditions in (33) are simplified to

$$\frac{\partial\phi}{\partial x} - \frac{\partial\psi}{\partial z} = \frac{\partial\phi}{\partial z} + \frac{\partial\psi}{\partial x} = 0, \quad \text{at} \quad z = 0. \tag{34}$$

In order to satisfy the above boundary conditions at the free surface $z = 0$, the following relation must be hold on $z = 0$:

$$k_1 \sin\theta_0 = k_1 \sin\theta_1 = k_2 \sin\theta_2 = k_3 \sin\theta_3, \tag{35}$$

which can also be written in the form

$$\theta_0 = \theta_1 \ \text{ and } \ \frac{\sin\theta_0}{V_1} = \frac{\sin\theta_2}{V_2} = \frac{\sin\theta_3}{V_3}. \tag{36}$$

In we neglect the thermal effect, then (36) reduces to

$$\theta_0 = \theta_1 \ \text{ and } \ \frac{\sin\theta_0}{\sqrt{c_p^2 - \varepsilon^2\omega^2}} = \frac{\sin\theta_3}{\sqrt{c_s^2 - \varepsilon^2\omega^2}}. \tag{37}$$

Further, in absence of the elastic nonlocality ($\varepsilon = 0$) from the medium, relation (37) becomes

$$\theta_0 = \theta_1 \ \text{ and } \ \frac{\sin\theta_0}{c_p} = \frac{\sin\theta_3}{c_s}, \tag{38}$$

which is the well-known *Snell's law*. We now consider the following two cases separately:

4.2 | Incident CP-wave at the thermally insulated rigid boundary

Inserting from Equations (29)–(31) into the boundary conditions (33) and (34) and using the relation (35), the following system of equations for the amplitude ratios, namely $R_{CP} = A_1/A_0$, $R_{CT} = A_2/A_0$, $R_{SV} = B_1/A_0$ of the reflected CP-, CT- and

SV-type waves, respectively are obtained:

$$\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & 0 \end{bmatrix} \begin{bmatrix} R_{CP} \\ R_{CT} \\ R_{SV} \end{bmatrix} = \begin{bmatrix} -a_{11} \\ a_{21} \\ a_{31} \end{bmatrix}, \tag{39}$$

where

$$a_{11} = k_1 \sin\theta_0, \quad a_{12} = k_2 \sin\theta_2, \quad a_{13} = -k_3 \cos\theta_3,$$
$$a_{21} = k_1 \cos\theta_0, \quad a_{22} = k_2 \cos\theta_2, \quad a_{23} = k_3 \sin\theta_3,$$
$$a_{31} = \zeta_1 k_1 \cos\theta_0, \quad a_{32} = \zeta_2 k_2 \cos\theta_2.$$

After solving (39), we may write the amplitude ratios in the following forms

$$R_{CP} = \frac{\zeta_1 \cos\theta_0 \cos(\theta_2 - \theta_3) - \zeta_2 \cos\theta_2 \cos(\theta_0 + \theta_3)}{\zeta_1 \cos\theta_0 \cos(\theta_2 - \theta_3) - \zeta_2 \cos\theta_2 \cos(\theta_0 - \theta_3)}, \tag{40}$$

$$R_{CT} = -\frac{\zeta_1 k_1 \sin 2\theta_0 \sin\theta_3 \sec\theta_2}{k_2 [\zeta_1 \cos\theta_0 (\cos\theta_3 + \sin\theta_3 \tan\theta_2) - \zeta_2 \cos(\theta_0 - \theta_3)]}, \tag{41}$$

$$R_{SV} = \frac{k_1 (\zeta_1 - \zeta_2) \sin 2\theta_0 \cos\theta_2 \csc\theta_3}{k_3 [\zeta_1 \cos\theta_0 (\sin\theta_2 + \cos\theta_2 \cot\theta_3) - \zeta_2 \cos\theta_2 (\sin\theta_0 + \cos\theta_0 \cot\theta_3)]}. \tag{42}$$

We note that, the amplitude ratios depend on the elastic properties of the medium, elastic nonlocality, and the angle of incidence.

In absence of the thermal field as well as the elastic nonlocality from the medium, Equations (20) and (38) give

$$c_1 = c_p, \quad c_2 = 0, \quad \theta_0 = \theta_1 \text{ and } \frac{\sin\theta_0}{c_p} = \frac{\sin\theta_3}{c_s}, \tag{43}$$

and the amplitude ratios reduce tow

$$R_{CP} = \frac{\cos(\theta_0 + \theta_3)}{\cos(\theta_0 - \theta_3)}, \quad R_{CT} = 0, \quad R_{SV} = \frac{\delta \sin 2\theta_0}{\cos(\theta_0 - \theta_3)}. \tag{44}$$

These expressions are in complete agreement with the corresponding results as reported by Achenberg [44] except the term δ in R_{SV} which is missing in Achenberg [44].

5 | ENERGY PARTITIONING

In order to physically justify the analytic expressions of the amplitude ratios in the present problem, we just need to verify the energy balance law at the boundary surface $z = 0$. Let us consider the energy partition between the various reflected waves at a surface element of the unit area. Following Achenberg [44], the energy flux across the surface element, that is, rate at which the energy is communicated per unit area of the surface is represented as

$$\bar{P} = \tau^L_{lm} \dot{u}_l n_m, \tag{45}$$

where n_m are the direction cosines of the unit normal outward to the surface element, and $\dot{u}_l$ are the components of the particle velocity. Here the summation convention is implied.

Note that here, the term corresponding to heat flux must be included in the expression of energy carried. Moreover, the interactional energy between different pairs of the waves must also be included. Now, in our present manuscript, we did not account these energies, because of following reasons: The contribution of interaction energy as well as of thermal energy is so small that even if it is accounted, it does not change the results qualitatively. This is our experience and we have seen it in other problems that these energies are just for the name sake. Their amount is of order of 10^{-5} for all angles of incidences. That is why the sum of the energy ratios does not change qualitatively. However, some physical situations may arise where the contribution of the thermal energy as well as the interaction energy is comparable to the other energies and in that cases it is essential to include these energies [cf. Li et al. [45, 46], Li and Wei [47]].

For the incident CP-wave, let $\langle P_0\rangle$ denotes the time-average stress power, $\langle P_1\rangle$ and $\langle P_2\rangle$, respectively denote the time-average stress power for the reflected CP- and CT-waves and $\langle P_3\rangle$ denotes the time-average stress power for the reflected SV-type wave.

We define the energy ratio E_α corresponding to the $\alpha-$th reflected wave on $z = 0$ as the ratio of the energy carried along with the $\alpha-$th reflected wave to the energy carried along the incident CP-wave i.e.,

$$E_\alpha = \frac{\langle P_\alpha\rangle}{\langle P_0\rangle}, \ \alpha = 1, 2, 3. \tag{46}$$

Thus, for the incident CP-wave, the analytical expressions for the energy ratios, E_{CP}, E_{CT} and E_{SV} of the reflected CP-, CT- and SV-type waves, respectively, may be obtained by using Equations (7), (8), (29)–(31) along with (45) and (46) as:

$$E_{CP} = -R_{CP}^2, \quad E_{CT} = -\frac{(k_2^2 + \zeta_2)\tan\theta_0}{(k_1^2 + \zeta_1)\tan\theta_2} R_{CT}^2, \quad E_{SV} = -\frac{\delta^2 k_3^2 \tan\theta_0}{(k_1^2 + \zeta_1)\tan\theta_3} R_{SV}^2, \tag{47}$$

where $0° < \theta_0 < 90°$. Like the amplitude ratios, the energy ratios also depend on θ_0, material properties of the nonlocal thermoelastic medium, and the amplitude ratios. Since, surface waves are not involved in the energy conservation principle, so the conservation of energy at the surface $z = 0$ may be stated as:

$$E_{sum} = |E_{CP} + E_{CT} + E_{SV}| \approx 1. \tag{48}$$

6 | NUMERICAL RESULTS AND DISCUSSIONS

With an aim to discuss the characteristics of plane harmonic wave propagation through a nonlocal thermoelastic material of type III, we have computed their phase speeds and the corresponding attenuation coefficients for a specific thermoelastic model of crust like material. The amplitude ratios and the corresponding energy ratios of the reflected CP-, CT- and SV-type waves due to the incident CP-wave have also been computed numerically and presented graphically to depict the effect of elastic nonlocality. The material chosen for this purpose is crust, whose material properties are read from Sarkar et al. [43] as follows:

$$\lambda = \mu = 3.0 \times 10^{10}\ N.m^{-2}, \quad T_0 = 300\ K, \quad \rho = 2900\ kg.m^{-3}, \quad C_E = 1100\ J.kg^{-1}.K^{-1}$$

$$K_T = 3.0\ W.m^{-1}.K^{-1}, \quad \alpha_T = 1.0667 \times 10^{-5}\ K^{-1}, \quad \varepsilon_\Theta = 0.00268, \quad \omega = 0.1.$$

Following [41], we have taken $v^2 = (\lambda + 2\mu)/\rho$, $K^* = C_E(\lambda + 2\mu)/4$, $c_p = 1.0$, and $c_t = 0.5$. Then we found $c_s = 0.4994$. Taking the constant $e_0 = 0.39$ [1] and choosing $a = 1$ nm, $l = 1$ nm, the the value of the nonlocal parameter, ε (dimensionless) can be obtained as, $\varepsilon = 2.3102$. The non-dimensional value of angular frequency is taken as $\omega = 0.1$.

Figure 2a-c depict the variations of the absolute values of amplitude ratios R_{CP}, R_{CT} and R_{SV} of the reflected CP-, CT- and the SV-type waves, respectively due to the incident CP-wave with respect to the angle of incidence θ_0 in the range ($0° \le \theta_0 \le 90°$) for nonlocal generalized thermoelasticity of type III (NGN III) and local generalized thermoelasticity of type III (LGN III). From Figure 2a, we conclude that the absolute values of R_{CP} are found to be decreasing for $0° \le \theta_0 \le 61°$) in NGN III, thereafter it increases as θ_0 increases. The maximum of $|R_{CP}|$ attains at $\theta_0 = 0°,\ 90°$ for both NGN III and LGN III cases. In Figure 2b, the amplitude ratio $|R_{CT}|$ is found to be larger for NGN III as compared to LGN III at each θ_0 in contrast to the amplitude ratio $|R_{SV}|$ (Figure 2c). It is also evident from Figures 2b and 2c that the maxima of $|R_{CT}|$ and $|R_{SV}|$, respectively occur at $\theta_0 = 65°$ and $\theta_0 = 45°$ for both types of NGN III and LGN III. Moreover, we see that $|R_{SV}|$ first increases in the range $0° \le \theta_0 \le 45°$ and then decreases for $45∘ < \theta_0 \le 90°$. Both the amplitude ratios R_{CT} and R_{SV} are zero at $\theta_0 = 0°,\ 90°$ for both types of NGN III and LGN III. Figure 2c also exhibits that $|R_{SV}|$ is symmetrical about $\theta_0 = 45°$. Figure 3a-c is plotted to examine how the amplitude ratios $|R_{CP}|$, $|R_{CT}|$ and $|R_{SV}|$ vary with the dimensionless nonlocal parameter ε. Here we choose the range of the dimensionless nonlocal parameter ε as $0 \le \varepsilon \le 2$ for fixed $\theta_0 = 45°$. We observe that the amplitude ratios $|R_{CP}|$ and $|R_{CT}|$ increase with the increase of ε for both the cases (NGN III and LGN III). On the contrary, the amplitude ratio $|R_{SV}|$ is found to be decreasing with increasing ε.

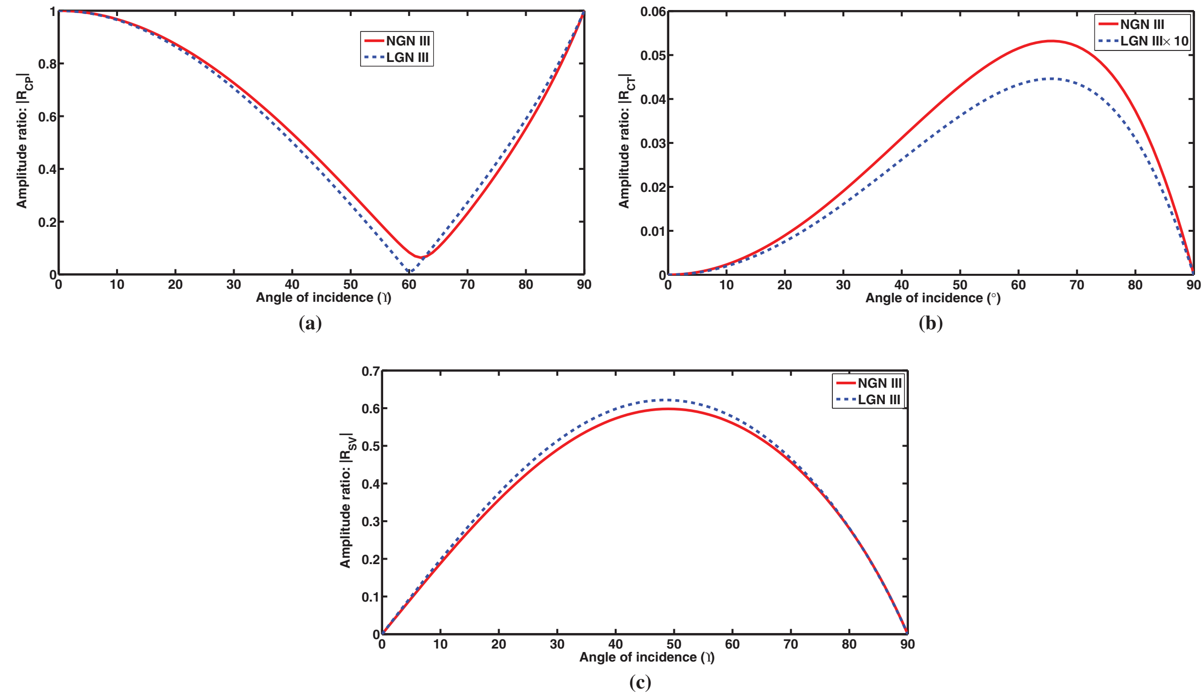

FIGURE 2 Variation of (a). $|R_{CP}|$, (b). $|R_{CT}|$, and (c). $|R_{SV}|$ with θ_0 for nonlocal Green-Nagdhi theory of type III (NGN III) and local Green-Nagdhi theory of type III (LGN III)

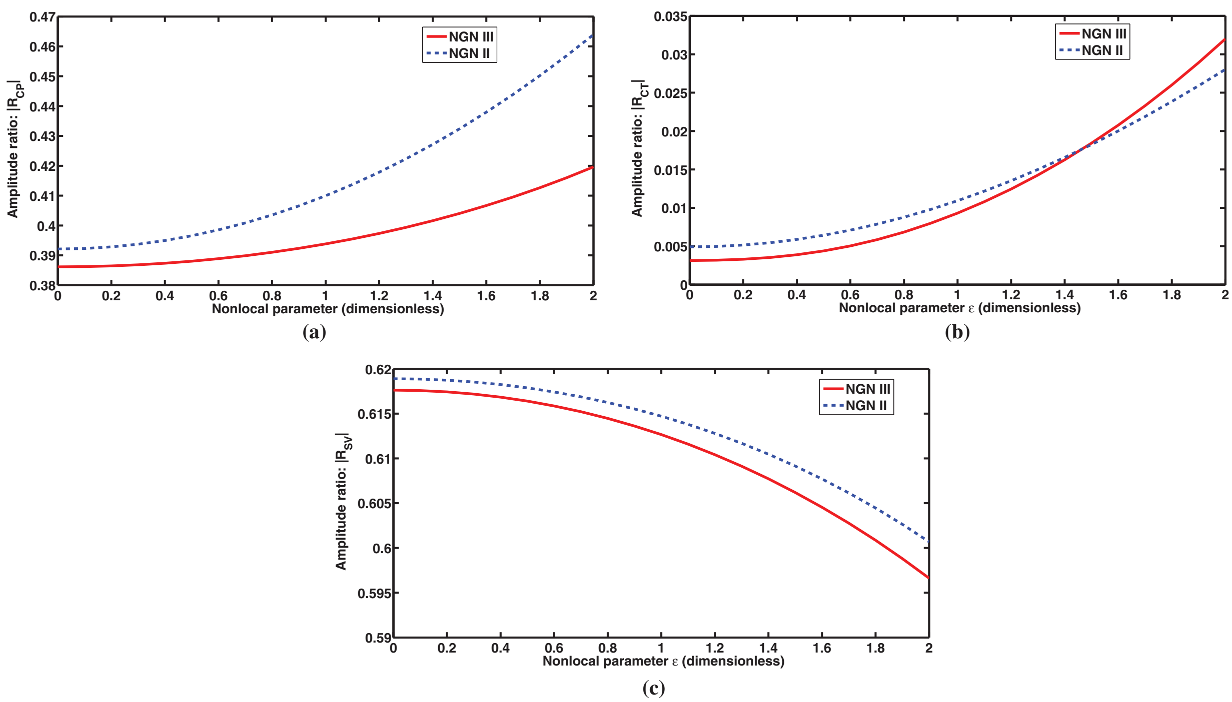

FIGURE 3 Variation of (a). $|R_{CP}|$, (b). $|R_{CT}|$, and (c). $|R_{SV}|$ with dimensionless elastic nonlocal parameter ε when $\theta_0 = 45°$

Figure 4a-c express a comparison of the energy ratios $|E_{CP}|$, $|E_{CT}|$ and $|E_{SV}|$ of the reflected CP-, CT- and SV-type waves, respectively due to the incident CP-wave with θ_0 for NGN III and LGN III cases. In Figure 4a, we notice that starting from the maximum at $\theta_0 = 0°$, the energy ratio $|E_{CP}|$ decreases and vanishes at $\theta_0 = 61°$ (approximately), then it increases to reach its maximum as θ_0 increases further in both the NGN III and LGN III. Figure 4b, c reveal the curves of the energy ratios $|E_{CT}|$ and $|E_{SV}|$ for NGN III are remaining upper than that in LGN III while the curve of $|E_{CP}|$ for NGN III is remaining upper than

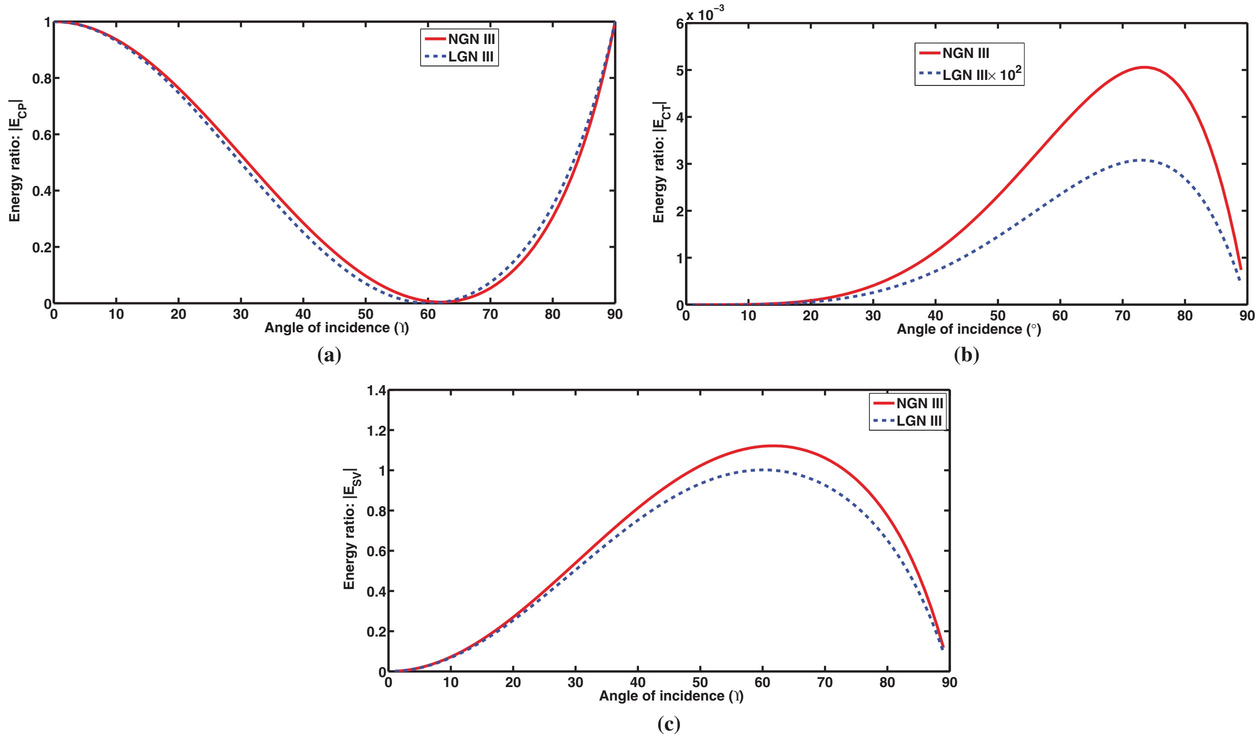

FIGURE 4 Variation of (a). $|E_{CP}|$, (b). $|E_{CT}|$, and (c). $|E_{SV}|$ with θ_0 for NGN III and LGN III

that for LGN III only in the range $0° \leq \theta_0 \leq 61°$. It is also evident that the profiles of the energy ratios with respect to θ_0 are qualitatively similar to the corresponding profiles of the amplitude ratios as shown in Figure 2a-c, apart from the magnitudes for both types of NGN III and LGN III. This is quite appealing as the energy ratios are proportional to the square of the corresponding amplitude ratios at each angle of incidence. The energy carried along the reflected CT-wave is the least which in turn means that the maximum amount of the incident energy is carried along the reflected CP- and the SV-type waves. The changes in the energy ratios with respect to ϵ are depicted through Figure 5a-c. It is revealed that the absolute values of energy ratios obtained for NGN III case are remaining smaller when compared to those obtained for the LGN III case. So, the presence of elastic nonlocality makes the energy ratios weaker.

In order to validate the numerical results about the reflected energy ratios, the energy conservation principle is checked up in Figure 6a-c in case of NGN III. The energy conservation principle requires that "the output energy fluxes are equal to the input energy flux at the same area" of the thermally insulated rigidly fixed surface $z = 0$. The energy ratio $|E_{CT}|$, carried along the reflected CT-wave, is plotted after mounting up its original value by 10^3 in Figure 6a. The curves indicated by "E_{sum}" in Figure 6a-c keep nearly unit value in the total range of θ_0, which means that the energy conservation law is satisfied at each θ_0. Hence, it is found that there is no dissipation of energy at the boundary surface $z = 0$ during the reflection of thermoelastic waves. However, Figure 6c shows a smaller deviation from the unity of the energy conservation index, E_{sum} which is attributed to the loss of numerical precision. The approximate satisfaction of the energy conservation law validates the present numerical results to a large extent. From Figure 6c, it is evident that for different values of the elastic nonlocality parameter ϵ, the energy balance law is also satisfied up to a large extent at a fixed angle of incidence.

To investigate the effect of the dimensionless elastic nonlocality parameter, ϵ on the existing coupled longitudinal and shear type waves in NGN III, we have plotted the phase speeds V_i $(i = 1, 2, 3)$ and the corresponding attenuation coefficients Q_i against ϵ in the range $0 \leq \epsilon \leq 2$ through Figure 7a-c. These figures clearly reveal that all the three phase speeds decrease when ϵ increases. It is also interesting to note that the CP- and CT-waves are dispersive and experience attenuation while though the SV-type wave is dispersive, it is not experiencing any attenuation in the selected range of ϵ. Figure 7a, b depict that the CP-wave is the fastest while SV-type wave is the slowest one.

The phase speed and the corresponding attenuation coefficient of the SV-type wave have been depicted in Figure 8. From this figure, we note that the SV-type wave is dispersive and non-attenuating in the range: $0.01 \leq \omega < 0.24991$, beyond which the wave is not a propagating wave. This is the verification of a result pointed out theoretically in the text. It can also be verified that $\omega \equiv \omega_{c_3} = 0.24991$ is correct.

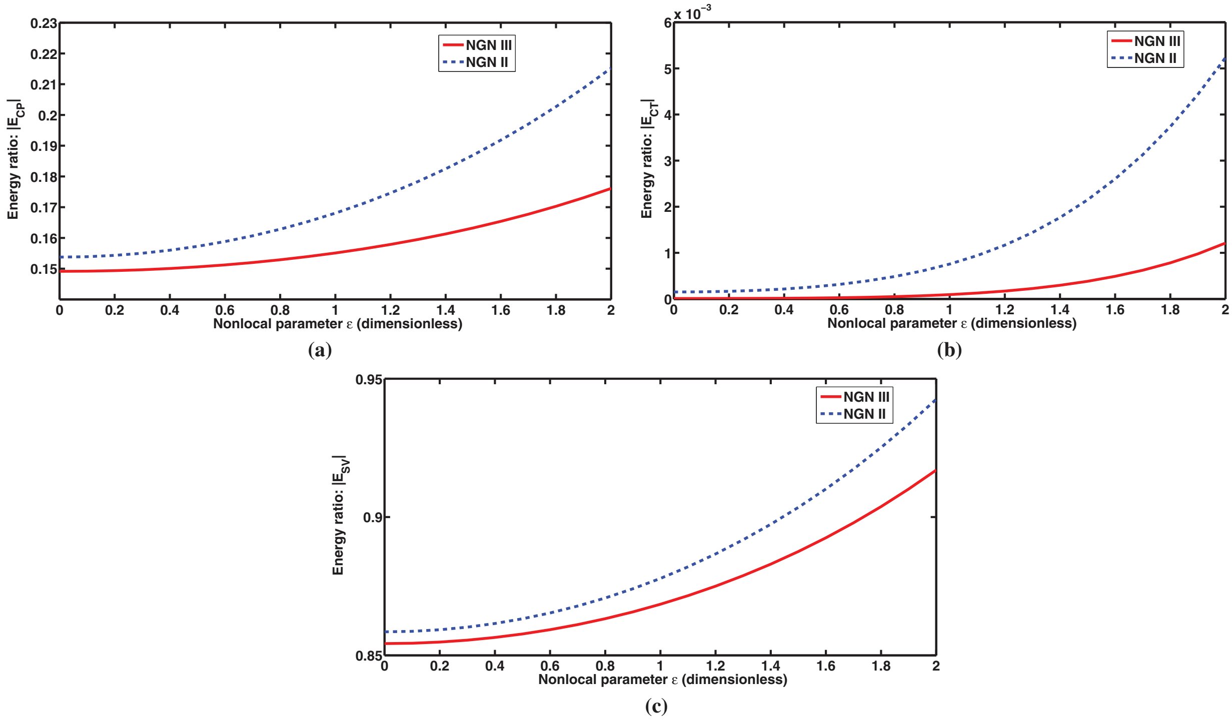

FIGURE 5 Variation of (a). $|E_{CP}|$, (b). $|E_{CT}|$, and (c). $|E_{SV}|$ with dimensionless elastic nonlocal parameter ε when $\theta_0 = 45°$

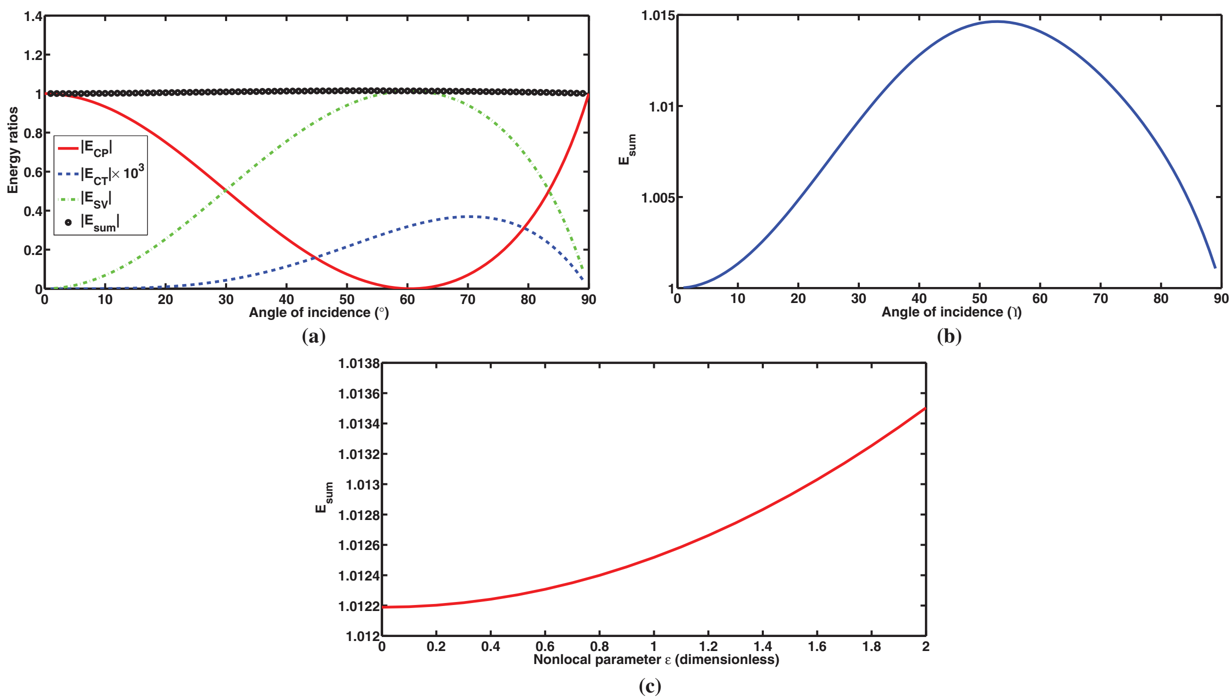

FIGURE 6 Variations of the energy ratios and their sum (E_{sum}) for NGN III

7 | CONCLUSIONS

This manuscript presents a mathematical treatment to discuss the harmonic plane waves in a thermoelastic medium by employing the Green-Naghdi theory of type III (GN model with energy dissipation) of generalized coupled thermoelasticity and Eringen's nonlocal theory of elasticity. The analytical expressions giving the reflection coefficients and the corresponding energy ratios

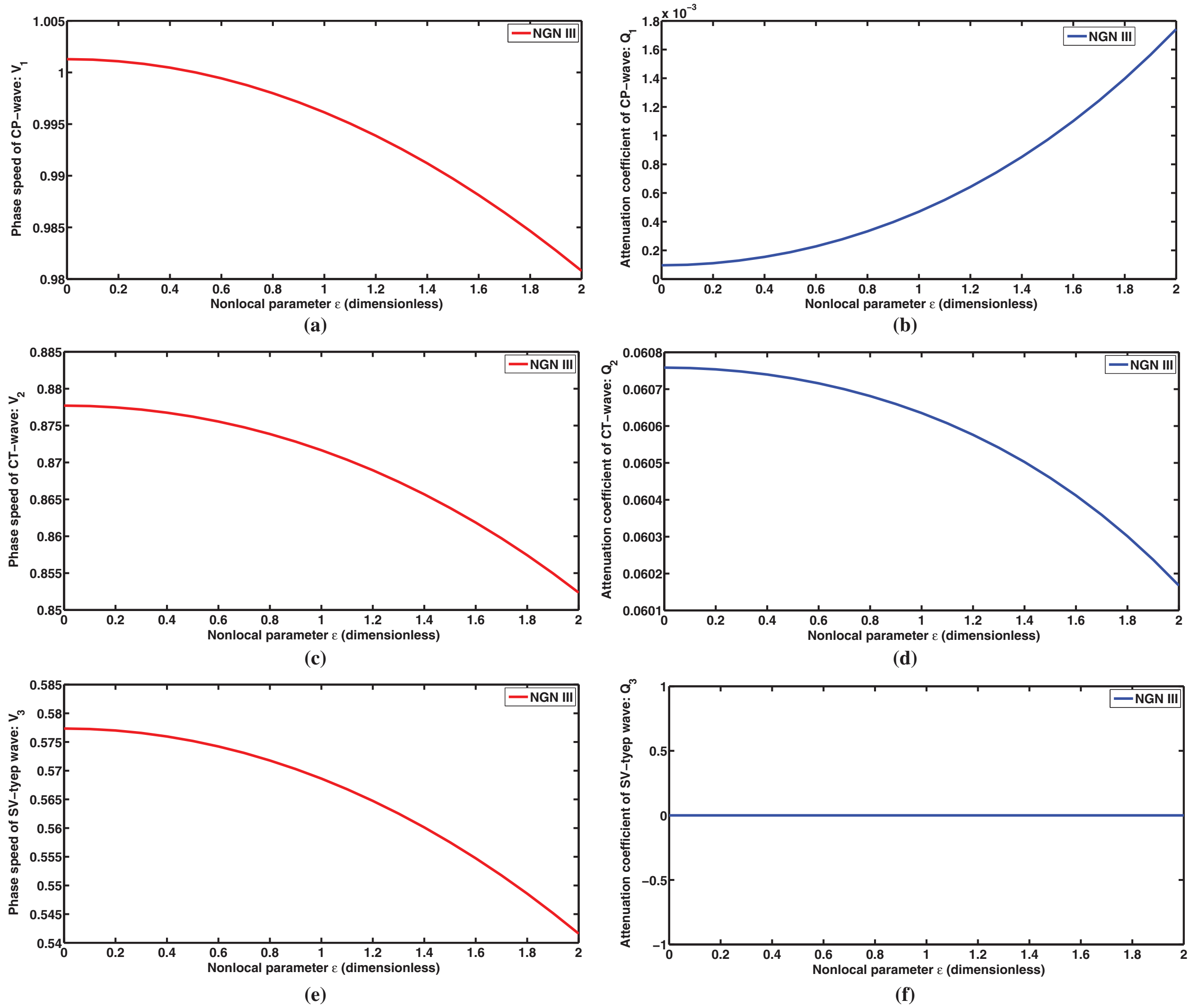

FIGURE 7 Variations of the phase speeds and the attenuations coefficient with dimensionless nonlocal elastic parameter ε when $\omega = 0.1$ (dimensionless)

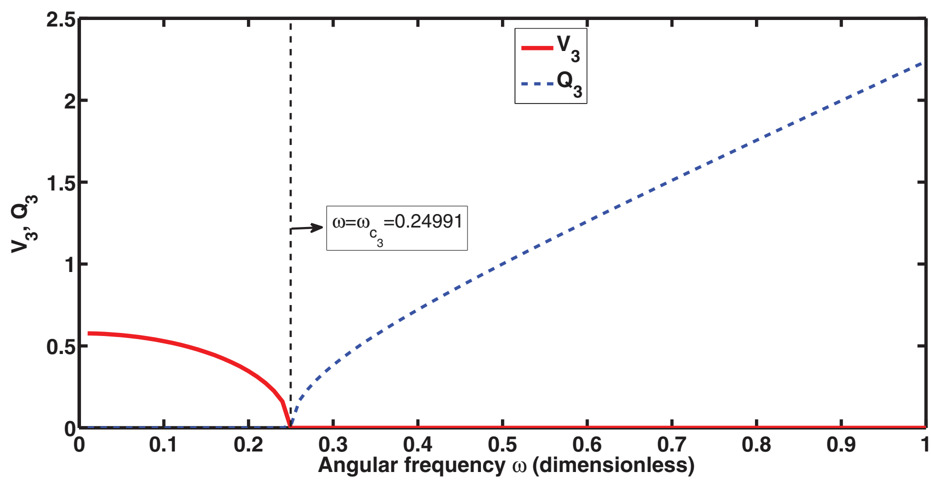

FIGURE 8 The phase speed (V_3) and attenuation coefficient (Q_3) of the SV-type wave versus non-dimensional angular frequency for NGN III

of various reflected waves due to the incident CP-wave waves are shown graphically. From the analysis of the illustrations, the following points can be noted:

(i) Similar to the local thermoelastic medium, three types of thermoelastic plane waves (CP-, CT- and SV-type) may propagate in a nonlocal thermoelastic medium with distinct phase speeds. The coupled longitudinal waves (CP- and CT-) are dispersive and experience no attenuation. The SV-type wave is also dispersive but exhibits no attenuation at low-frequency range due to the presence of the elastic nonlocality. Moreover, the SV-type wave is unaffected by the thermal wave effect in contrast to the CP- and CT-waves.

(ii) The presence of elastic nonlocality reduces the classical vertically shear (SV) wave speed. The SV-type wave faces a critical frequency in the nonlocal medium considered.

(iii) The numerical results display that the phase speeds, attenuation coefficients, amplitude ratios and the energy ratios of various reflected waves are significantly affected by the elastic nonlocality.

(iv) It is observed from the numerical results that the maximum amount of the incident energy is carried along the reflected CP-wave and SV-type wave. It is also evident that the sum of the modulus values of the energy ratios is approximately unity at each angle of incidence which in turn means that the energy conservation law is satisfied during the reflection of the thermoelastic waves in a nonlocal thermoelastic medium.

(v) The introduction of the elastic nonlocality into the generalized thermoelasticity gives a more realistic model for the study of harmonic plane waves in thermoelastic solids. The authors believe that the present theoretical investigation may provide some interesting information for experimental scientists/ researchers/seismologists working on this subject.

ACKNOWLEDGMENTS

The authors would like to thank the Editor and the anonymous referee for their suggestions and comments to improve the manuscript.

DISCLOSURE

The authors declare that they have no conflict of interest.

ORCID

Nantu Sarkar https://orcid.org/0000-0001-9144-4587

REFERENCES

[1] A. C. Eringen, *Nonlocal Continuum Field Theories*, Springer, New York **2002**.
[2] D. G. B. Edelen, N. Laws, On the thermodynamics of systems with nonlocality, *Arch. Ration. Mech. Anal.* **1971**, *43*, 24.
[3] D. G. B. Edelen, A. E. Green, N. Lows, Nonlocal continuum mechanics, *Arch. Ration. Mech. Anal.* **1971**, *43*, 36.
[4] A. C. Eringen, D. G. B. Edelen, On nonlocal elasticity, *Int. J. Eng. Sci.* **1972**, *10*, 233.
[5] B. S. Altan, Uniqueness in the linear theory of nonlocal elasticity, *Bull. Tech. Univ. Istanb.* **1984**, *37*, 373.
[6] S. Chirita, On some boundary value problems in nonlocal elasticity, *Amale Stiinfice ale Universitatii "AL. I. CUZA" din Iasi Tomul*, Vol. xxii **1976**, p. 2.
[7] S. Narendra, S. Gopalakrishnan, Ultrasonic wave characteristics of nanorods via nonlocal strain gradient models, *J. Appl. Phys.* **2010**, *107*, 084312.
[8] S. Narendar, S. Gopalakrishnan, Nonlocal scale effects of ultrasonic wave characteristics of nanorods, *Physica E* **2010**, *42*, 1601.
[9] S. Narendar, D. R. Mahapatra, S. Gopalakrishnan, Prediction of nonlocal scaling parameter for armchair and zigzag single-walled carbon nanotubes based on molecular structural mechanics, nonlocal elasticity and wave propagation, *Int. J. Eng. Sci.* **2011**, *49*, 509.
[10] S. Narendra, Spectral finite element and nonlocal continuum mechanics based formulation for tortional wave propagation in nanorods, *Finite Elem. Anal. Des.* **2012**, *62*, 65.
[11] M. Malagu, E. Benvenuti, A. Simone, One-dimensional nonlocal elasticity for tensile single-walled carbon nanotubes:a molecular structural mechanics characterization, *Eur. J. Mech. A, Solids* **2015**, *54*, 160.
[12] A. Khurana, S. K. Tomar, Wave propagation in nonlocal microstretch solid, *Appl. Math. Model.* **2016**, *40*, 5858.
[13] D. Sing, G. Kaur, S. K. Tomar, Waves in nonlocal elastic solid with voids, *J. Elast.* **2017**, *128*, 85.
[14] A. C. Eringen, Theory of nonlocal thermoelasticity, *Int. J. Eng. Sci.* **1974**, *12*, 1063.
[15] F. Balta, E. S. Suhubi, Theory of nonlocal generalized thermoelasticity, *Int. J. Eng. Sci.* **1977**, *15*, 577.

[16] Y. J. Yu, X-G. Tian, X-R. Liu, Size-dependent generalized thermoelasticity using Eringen's nonlocal model, *Eur. J. Mech. A, Solids* **2015**, *51*, 96.

[17] Y. J. Yu, X-G. Tian, X-R. Liu, Nonlocal thermoelasticity based on nonlocal heat conduction and nonlocal elasticity, *Eur. J. Mech. A, Solids* **2016**, *60*, 238.

[18] M. Bachher, N. Sarkar, Nonlocal theory of thermoelastic materials with voids and fractional derivative heat transfer, *Waves in Random and Complex Media* **2019**, *29*, 595.

[19] Y. Li, W. Wang, P. Wei, C. Wang, Reflection and transmission of elastic waves at an interface with consideration of couple stress and thermal wave effects, *Meccanica* **2018**, *53*, 2921.

[20] N. Sarkar, S. K. Tomar, Plane waves in nonlocal thermoelasticsolid with voids, *J. Therm. Stresses* **2019**, *42*, 580.

[21] S. Mondal, N. Sarkar, N. Sarkar, Waves in dual-phase-lag thermoelastic materials with voids based on Eringen's nonlocal elasticity, *J. Therm. Stresses* **2019**, *42*, 1035.

[22] H. Jeffreys, The thermodynamics of an elastic solid, *Proc. Camb. Philos. Soc.* **1930**, *26*, 101.

[23] B. Gutenberg, Energy relation of reflected and refracted seismic waves, *Seismol. Soc. Am.* **1944**, *34*, 85.

[24] C. G. Knott, Reflection and refraction of elastic waves with seismological applications, *Philos. Mag.* **1899**, *48*, 64.

[25] C. E. Beevers, J. Bree, A note on wave reflection problems in linear thermoelasticity, *J. Mech. Phys. Solids* **1975**, *9*, 355.

[26] A. N. Sinha, S. B. Sinha, Reflection of thermoelastic waves at a solid half-space with thermal telaxation, *J. Phys. Earth* **1974**, *22*, 237.

[27] S. B. Sinha, K. A. Elsibai, Reflection and reflection of thermoelastic waves at an interface of two semi-infinite media with two relaxation times, *J. Therm. Stresses* **1997**, *20*, 129.

[28] J. N. Sharma, V. Kumar, D. Chand, Reflection of generalized thermoelastic waves from the boundary of half space, *J. Therm. Stresses* **2003**, *26*, 925.

[29] A. E. Green, P. M. Naghdi, Thermoelasticity without energy disspation, *J. Elast.* **1993**, *31*, 189.

[30] H. W. Lord, Y. A. Shulman, Generalized dynamical theory of thermoelasticity, *J. Mech. Phys. Solids* **1967**, *15*, 229.

[31] A. E. Green, K. A. Lindsay, Thermoelasticity, *J. Elast.* **1972**, *2*, 1.

[32] N. C. Das, A. Lahiri, S. Sarkar, S. Basu, Reflection of generalized thermoelastic waves from isothermal and insulated boundaries of a half space, *Comput. Math. Appl.* **2008**, *56*, 2795.

[33] R. Kumar, N. Sharma, P. Lata, Thermomechanical interactions in a transversely isotropic magnetothermoelastic with and without energy dissipation with combined effects of rotation, vacuum and two temperatures, *Applied Mathmatical Modelling* **2016**, *40*, 6560.

[34] P. Lata, Reflection and refraction of plane waves in layered nonlocal elastic and anisotropic thermoelastic medium, *Struct. Eng. Mech.* **2018**, *66*, 113.

[35] P. Lata, Effect of energy dissipation on plane waves in sandwiched layered thermoelastic medium, *Steel and Composite Structures* **2018**, *27*, 439.

[36] M. I. A. Othman, Y. Q. Song, The effect of rotation on the reflection of magnetothermoelastic waves under thermoelasticity without energy dissipation, *Acta Mech.* **2006**, *184*, 189.

[37] M. I. A. Othman, Y. Q. Song, Reflection of plane waves from an elastic solid half-space under hydrostatic initial stress without energy dissipation, *Int. J. Solids Struct.* **2007**, *44*, 5651.

[38] M. I. A. Othman, Y. Q. Song, Reflection of magneto-thermoelasticity waves with two relaxation times and temperature dependent elastic moduli, *Appl. Math. Model.* **2008**, *32*, 483.

[39] M. I. A. Othman, Y. Q. Song, Reflection of magneto-thermoelastic waves from a rotating elastic half-space, *Int. J. Eng. Sci.* **2008**, *46*, 459.

[40] A. E. Green, P. M. Naghdi, On undamped heat waves in an elastic solid, *J. Therm. Stresses* **1992**, *15*, 253.

[41] D. S. Chandrasekharaiah, K. S. Srinath, Thermoelastic plane waves without energy dissipation in a rotating body, *Mech. Res. Commun.* **1997**, *24*, 551.

[42] N. Das, N. Sarkar, A. Lahiri, Reflection of plane waves from the stress-free isothermal and insulated boundaries of a nonlocal thermoelastic solid, *Applied Mathmatical Modelling* **2019**, *73*, 526.

[43] N. Sarkar, S. De, N. Sarkar, Waves in nonlocal thermoelastic solids of type II, *J. Therm. Stresses* **2019**, *42*, 1153.

[44] J. D. Achenbach, *Wave Propagation in Elastic Solids*, North-Holland, New York **1976**.

[45] Y. Li, P. Wei, C. Wang, Propagation of thermoelastic waves across an interface with consideration of couple stress and second sound, *Math. Mech. Solids* **2019**, *24*, 235.

[46] Y. Li, L. Li, P. Wei, Reflection and refraction of thermoelastic waves at an interface of two couple-stress solids based on Lord-Shulman thermoelastic theory, *Appl. Math. Model.* **2018**, *55*, 536.

[47] Y. Li, P. Wei, Reflection and transmission of thermo-elastic waves without energy dissipation at the interface of two dipolar gradient elastic solids, *J. Acoust. Soc. Am.* **2018**, *143*, 550.

How to cite this article: N. Sarkar, M. Bachher, N. Das, S. De, N. Sarkar. Waves in nonlocal thermoelastic solids of type III. *Z Angew Math Mech*. 2020;e201900074. https://doi.org/10.1002/zamm.201900074